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DOUBLE $(r, s)(u, v)$-PREOPEN SETS

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Abstract. We introduce the concepts of double $(r, s)(u, v)$-preopen sets, double $(r, s)(u, v)$-preclosed sets and double pairwise $(r, s)(u, v)$-precontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

1. Introduction

Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and M. Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double $(r, s)(u, v)$-preopen sets, double $(r, s)(u, v)$-preclosed sets and double pairwise $(r, s)(u, v)$-precontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$.

DEFINITION 2.1. [1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X,T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I\otimes I$ be the set of the pair (r,s) such that $r,s\in I$ and $r+s\leq 1$.

DEFINITION 2.2. [12] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma}=(\mathcal{T}^{\mu},\mathcal{T}^{\gamma})$ on X is a mapping $\mathcal{T}^{\mu\gamma}: I(X)\to I\otimes I(\mathcal{T}^{\mu},\mathcal{T}^{\gamma}: I(X)\to I)$ which satisfies the following properties:

(1) $\mathcal{T}^{\mu}(0_{\sim})=\mathcal{T}^{\mu}(1_{\sim})=1$ and $\mathcal{T}^{\gamma}(0_{\sim})=\mathcal{T}^{\gamma}(1_{\sim})=0$.
(2) $\mathcal{T}^{\mu}(A\cap B)\geq\mathcal{T}^{\mu}(A)\wedge\mathcal{T}^{\mu}(B)$ and $\mathcal{T}^{\gamma}(A\cap B)\leq\mathcal{T}^{\gamma}(A)\vee\mathcal{T}^{\gamma}(B)$.
(3) $\mathcal{T}^{\mu}(\bigcup A_i)\geq\bigwedge\mathcal{T}^{\mu}(A_i)$ and $\mathcal{T}^{\gamma}(\bigcup A_i)\leq\bigvee\mathcal{T}^{\gamma}(A_i)$.

The $(X,\mathcal{T}^{\mu\gamma})=(X,\mathcal{T}^{\mu},\mathcal{T}^{\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense.* Also, we call $\mathcal{T}^{\mu}(A)$ a *gradation of openness* of A and $\mathcal{T}^{\gamma}(A)$ a *gradation of nonopenness* of A.

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X,\mathcal{T}^{\mu},\mathcal{T}^{\gamma})$ and $(r,s)\in I\otimes I$. Then A is said to be

(1) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-open* set if $\mathcal{T}^{\mu}(A)\geq r$ and $\mathcal{T}^{\gamma}(A)\leq s$,
(2) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-closed* set if $\mathcal{T}^{\mu}(A^c)\geq r$ and $\mathcal{T}^{\gamma}(A^c)\leq s$.

Let $(X,\mathcal{T}^{\mu},\mathcal{T}^{\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r,s)\in I\otimes I$ and for each $A\in I(X)$, the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)=\bigcap\{B\in I(X)\mid A\subseteq B, B\text{ is }\mathcal{T}^{\mu\gamma}\text{-fuzzy }(r,s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)=\bigcup\{B\in I(X)\mid A\supseteq B, B\text{ is }\mathcal{T}^{\mu\gamma}\text{-fuzzy }(r,s)\text{-open}\}.$$

LEMMA 2.3. [9] For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak's sense $(X,\mathcal{T}^{\mu},\mathcal{T}^{\gamma})$ and $(r,s)\in I\otimes I$, we have:

(1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)^c=\mathcal{T}^{\mu\gamma}\text{-int}(A^c,r,s)$.
(2) $\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)^c=\mathcal{T}^{\mu\gamma}\text{-cl}(A^c,r,s)$.

A system $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space.*

DEFINITION 2.4. [9] Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$. Then A is said to be

(1) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B,u,v)$,
(2) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen if there is an $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B in X such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(B,r,s)$,
(3) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set B in X such that $\mathcal{U}^{\mu\gamma}\text{-int}(B,u,v) \subseteq A \subseteq B$,
(4) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed if there is an $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set B in X such that $\mathcal{T}^{\mu\gamma}\text{-int}(B,r,s) \subseteq A \subseteq B$.

3. Double $(r,s)(u,v)$-preopen sets

Definition 3.1. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then A is said to be

(1) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preopen* set if
$$A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v),r,s),$$
(2) an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preopen* set if
$$A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s),u,v),$$
(3) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preclosed* set if
$$A \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}((\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v),r,s),$$
(4) an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preclosed* set if
$$A \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}((\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s),u,v).$$

Theorem 3.2. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.
(2) A^c is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set.

Proof. It follows from Lemma 2.3. □

Corollary 3.3. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) A is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.
(2) A^c is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set.

Theorem 3.4. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$.

(1) If A is $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open of $(X,\mathcal{T}^{\mu\gamma})$, then A is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

(2) If A is $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed of $(X,\mathcal{T}^{\mu\gamma})$, then A is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

Proof. (1) Let A be a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set of $(X,\mathcal{T}^{\mu\gamma})$. Then $A=\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)$. Clearly, we have $A\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v)$ and hence

$$A=\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v),r,s).$$

Thus A is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

(2) Let A be a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set of $(X,\mathcal{T}^{\mu\gamma})$. Then $A=\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)$. Clearly, we have $A\supseteq\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v)$ and hence

$$A=\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)\supseteq\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v),r,s).$$

Thus A is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$. □

Corollary 3.5. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$.

(1) If A is $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open of $(X,\mathcal{U}^{\mu\gamma})$, then A is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.
(2) If A is $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed of $(X,\mathcal{U}^{\mu\gamma})$, then A is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

But the converses in the above theorem and corollary need not be true which is shown by the following example.

Example 3.6. Let $X=\{x,y\}$ and let A_1,A_2,A_3 and A_4 be intuitionistic fuzzy sets of X defined as

$$A_1(x)=(0.2,0.4),\quad A_1(y)=(0.6,0.3);$$
$$A_2(x)=(0.4,0.3),\quad A_2(y)=(0.7,0.1);$$
$$A_3(x)=(0.5,0.2),\quad A_3(y)=(0.1,0.8);$$

and

$$A_4(x)=(0.5,0.3),\quad A_4(y)=(0.2,0.4).$$

Define $\mathcal{T}^{\mu\gamma}:I(X)\to I\otimes I$ and $\mathcal{U}^{\mu\gamma}:I(X)\to I\otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A)=(\mathcal{T}^{\mu}(A),\mathcal{T}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{5}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise;}\end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A)=(\mathcal{U}^{\mu}(A),\mathcal{U}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{3},\frac{1}{4}) & \text{if } A=A_2,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-open set and A_3^c is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preclosed set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-closed set. Also A_4 is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preopen set which is not an $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-open set and A_4^c is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preclosed set which is not an $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-closed set.

Lemma 3.7. That $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen ($(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen) and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen ($(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen) are independent notions is shown by the following example.

Example 3.8. Let $X = \{x, y\}$ and let A_1, A_2, A_3, A_4, A_5 and A_6 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.2, 0.7), \quad A_1(y) = (0.5, 0.3);$$
$$A_2(x) = (0.5, 0.4), \quad A_2(y) = (0.2, 0.6);$$
$$A_3(x) = (0.5, 0.3), \quad A_3(y) = (0.4, 0.2);$$
$$A_4(x) = (0.3, 0.6), \quad A_4(y) = (0.5, 0.2);$$
$$A_5(x) = (0.8, 0.1), \quad A_5(y) = (0.1, 0.7);$$

and

$$A_6(x) = (0.6, 0.2), \quad A_6(y) = (0.2, 0.5).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set and A_4 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set. Also A_5 is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})$ $(\frac{1}{2}, \frac{1}{5})$-preopen set which is not an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiopen set and A_6 is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiopen set which is not an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-fuzzy $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preopen set.

THEOREM 3.9. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v)\in I\otimes I$.

(1) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen sets of X, then $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.

(2) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen sets of X, then $\bigcup A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.

(3) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed sets of X, then $\bigcap A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set.

(4) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed sets of X, then $\bigcap A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set.

Proof. (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen sets. Then for each k, $A_k\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A_k,u,v),r,s)$. So

$$\begin{aligned}\bigcup A_k &\subseteq \bigcup \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A_k,u,v),r,s)\\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\bigcup A_k,u,v),r,s).\end{aligned}$$

Thus $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen sets. Then for each k, $A_k\subseteq\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A_k,r,s),u,v)$. So

$$\begin{aligned}\bigcup A_k &\subseteq \bigcup \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A_k,r,s),u,v)\\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\bigcup A_k,r,s),u,v).\end{aligned}$$

Thus $\bigcup A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.

(3) It follows from (1) using Theorem 3.2.

(4) It follows from (2) using Corollary 3.3 □

DEFINITION 3.10. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s),(u,v)\in I\otimes I$. Then f is called *double pairwise* $(r,s)(u,v)$-*precontinuous* if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set A of Y and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set B of Y.

THEOREM 3.11. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent:

(1) f is a double pairwise $(r,s)(u,v)$-precontinuous mapping.

(2) $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set A of Y and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set B of Y.

(3) For each intuitionistic fuzzy set A of Y,

$$f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)) \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)$$

and

$$f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)) \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(A),r,s),u,v).$$

(4) For each intuitionistic fuzzy set C of X,

$$\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s) \supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s)$$

and

$$\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v) \supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v).$$

Proof. (1) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then A^c is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B^c is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Since f is double pairwise $(r,s)(u,v)$-precontinuous, $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set and $f^{-1}(B^c)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of X. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X.

(2) $\Rightarrow$ (1) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Then A^c is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B^c is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B^c)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of X. Thus f is a double pairwise $(r,s)(u,v)$-precontinuous mapping.

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set of Y. Then $\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v))$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X. Thus

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)) &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)),u,v),r,s) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)) &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)),r,s),u,v) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(A),r,s),u,v). \end{aligned}$$

(3) $\Rightarrow$ (4) Let C be any intuitionistic fuzzy set of X. Then $f(C)$ is an intuitionistic fuzzy set of Y. By (3),

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s)) &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}f(C),u,v),r,s) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v)) &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}f(C),r,s),u,v) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v). \end{aligned}$$

Hence

$$\begin{aligned} \mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s) &\supseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s)) \\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s)) \end{aligned}$$

and

$$\begin{aligned} \mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v) &\supseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v)) \\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v)). \end{aligned}$$

(4) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of X. By (4),

$$\begin{aligned} A = \mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s) &\supseteq \mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A),r,s) \\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)) \end{aligned}$$

and

$$\begin{aligned} B = \mathcal{W}^{\mu\gamma}\text{-cl}(B,u,v) &\supseteq \mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(B),u,v) \\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v)). \end{aligned}$$

So

$$\begin{aligned} f^{-1}(A) &\supseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(B) &\supseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v)) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v). \end{aligned}$$

Therefore $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X. $\square$

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